Unitarity

and

Bounds on the scale of Fermion Mass Generation

in

Deconstructed Higgsless Models

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Preview: AC

What is the scale of fermion mass generation?

- Is it the same as the scale of EW gauge boson mass generation?

Can we find an upper bound on this scale?

- Yes – PRL **59**, 2405 (1987)

  Appelquist and Chanowitz did this for the SM without a Higgs.
- Unitarity breaks down in the process $t\bar{t} \to W_L^+ W_L^-$ if new physics does not appear before Λ_{AC} .

Is the AC bound truly independent?

- M. Golden: PLB **338**, 295 (1994)

  Won't the fields that unitarize WW scattering also unitarize $t\bar{t} \to W_L^+ W_L^-$?
- In the SM, the Higgs unitarizes both.
- In Higgsless models, distinct fields unitarize $t\bar{t} \to W_L^+ W_L^-$ and WW scattering.

Is the 2->m process stronger?

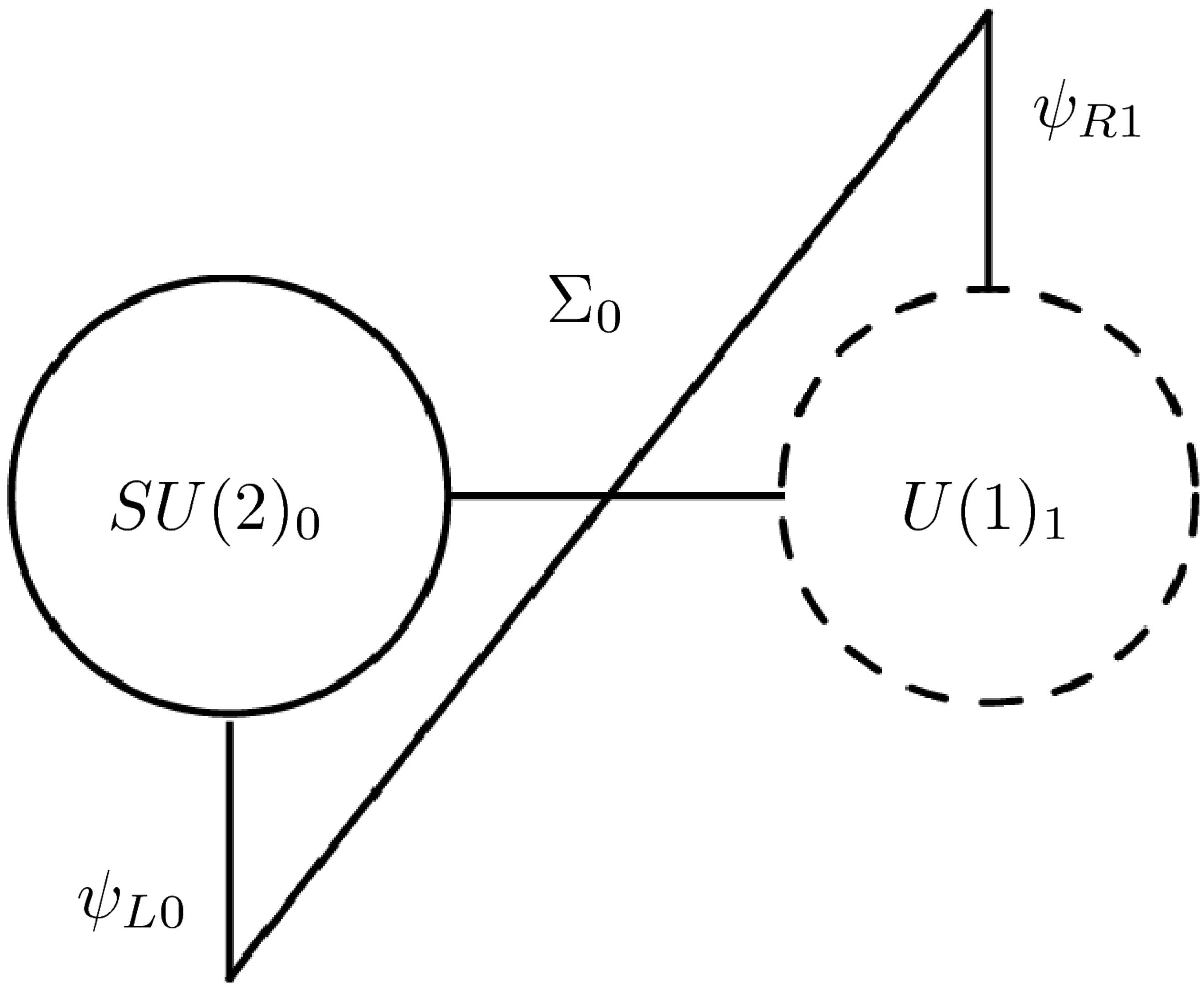
- PRD **65**, 033004 (2002)
  Maltoni, Niczyporuk,
  and Willenbrock noted that the 2->m
  process can sometimes give a
  stronger bound.
- PRD **71**, 093009 (2005)
  Dicus and He showed that for the top
  quark, the 2->2 process was still the
  strongest.

Preview: CCCS

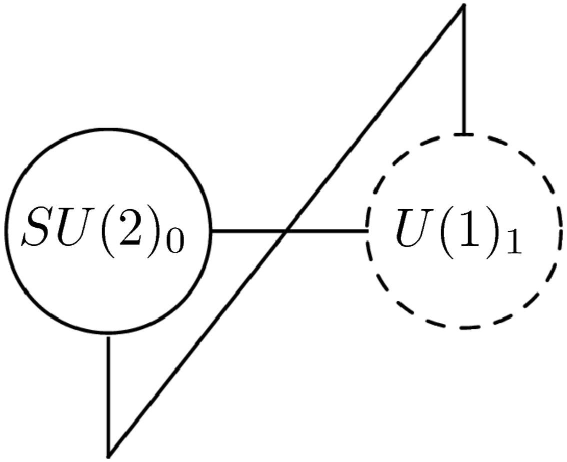
How is this scale modified in Higgsless models?

- PRD **75**, 073018 (2007)
  Chivukula, Christensen, Coleppa and Simmons showed that $t\bar{t} \rightarrow W_L^+ W_L^-$ is unitarized by a set of fields distinct from those which unitarize WW scattering.
- The scale where unitarity breaks down is a function of the mass of the 1st KK mode of the fermions and is independent of the mass of the 1st KK mode of the gauge bosons.

2-Site Model: (Higgsless SM)



2-Site Model: Gauge



$$\mathcal{L}_{F^2} = -\frac{1}{2}\mathrm{Tr}\left[F_0^2 + F_1^2\right]$$

where

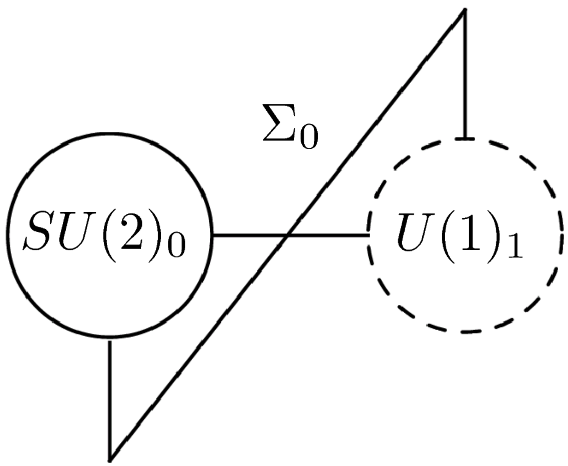
$$F_0^{\mu\nu} = \partial^\mu W_0^\mu - \partial^\nu W_0^\mu + ig\left[W_0^\mu, W_0^\nu\right]$$

$$F_1^{\mu\nu} = \partial^\mu W_1^\mu - \partial^\nu W_1^\mu$$

$$W_0 = \begin{pmatrix} \frac{1}{2}W_0^0 & \frac{1}{\sqrt{2}}W_0^+ \\ \frac{1}{\sqrt{2}}W_0^- & -\frac{1}{2}W_0^0 \end{pmatrix}$$

$$W_1 = \begin{pmatrix} \frac{1}{2}W_1^0 & 0 \\ 0 & -\frac{1}{2}W_1^0 \end{pmatrix}$$

2-Site Model: Gauge-Goldstone



$$\mathcal{L}_{D\Sigma} = \frac{f^2}{2}\mathrm{Tr}\Big[\,(D_\mu\Sigma_0)^\dagger\, D^\mu\Sigma_0\Big]$$

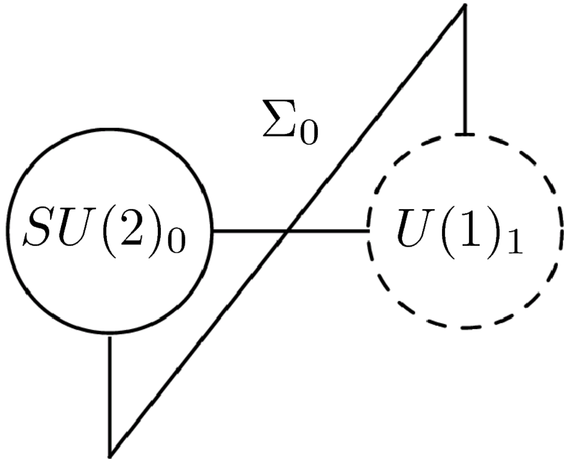
where

$$D_\mu\Sigma_0 = \partial_\mu\Sigma_0 + igW_0\Sigma_0 - ig'\Sigma_0 W_1$$

$$\Sigma_0 = e^{i\frac{2\pi_0}{f}}$$

$$\pi_0 = \begin{pmatrix} \frac{1}{2}\pi_0^0 & \frac{1}{\sqrt{2}}\pi_0^+ \\ \frac{1}{\sqrt{2}}\pi_0^- & -\frac{1}{2}\pi_0^0 \end{pmatrix}$$

2-Site Model: Gauge Masses



$$\mathcal{L}_{WW} = \frac{f^2}{2}\mathrm{Tr}\Big[\,(D_\mu I)^\dagger\, D^\mu I\Big]$$

$$M_n^2 = \frac{f^2}{4}\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}$$

$$M_\pm^2 = \frac{f^2}{4}\left(g^2\right)$$

$$M_\gamma^2 = 0 \qquad v_\gamma = e\left\{\frac{1}{g}\,,\;\frac{1}{g'}\right\}$$

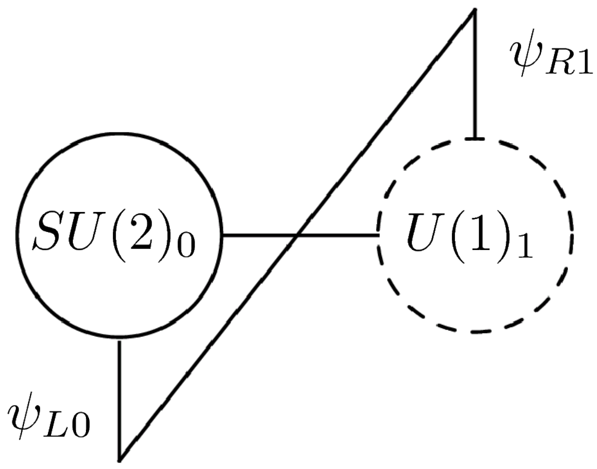
$$M_W^2 = \frac{g^2 f^2}{4}$$

$$M_Z^2 = \frac{\left(g^2 + g'^2\right) f^2}{4}$$

$$v_W = \{1\}$$

$$v_Z = e\left\{\frac{1}{g'}\,,\;-\frac{1}{g}\right\}$$

2-Site Model: Fermion-Gauge



$$\mathcal{L}_{D\psi} = \bar{\psi}_{L0} \not{D}\, \psi_{L0} + \bar{\psi}_{R1} \not{D}\, \psi_{R1}$$

where

$$D_\mu \psi_{L0} = \partial_\mu \psi_{L0} + igW_0\psi_{L0} + ig' Y_{0f} W_1 \psi_{L0}$$

$$D_\mu \psi_{R1} = \partial_\mu \psi_{R1} + ig' Y_{1f} W_1 \psi_{R1}$$

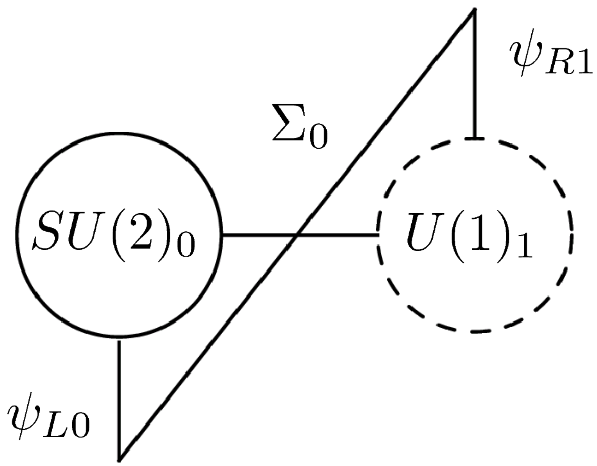
$Y_{1u} = 4/3$

$Y_{0Q} = 1/3$

$Y_{1d} = -2/3$

$Y_{0L} = -1$ $\quad Y_{1e} = -2$

2-Site Model: Fermion-Goldstone



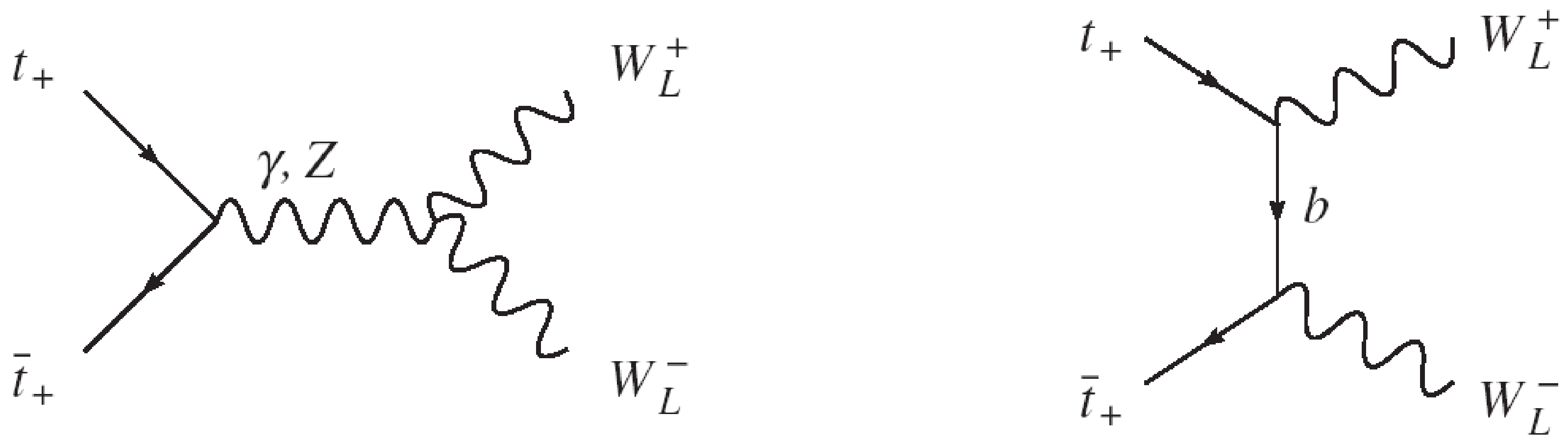
$$\mathcal{L} = -M_F \epsilon_{Rf} \bar{\psi}_{L0} \Sigma_0 \psi_{R1}$$

$$m_f = M_F \epsilon_{Rf}$$

$$v_L = \{1\}$$

$$v_R = \{1\}$$

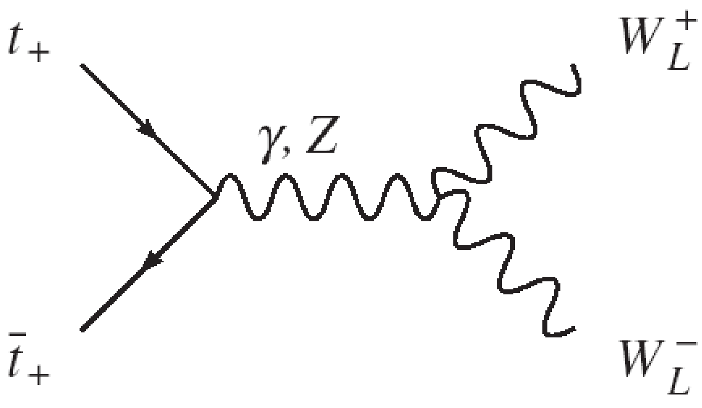
AC Bound 1

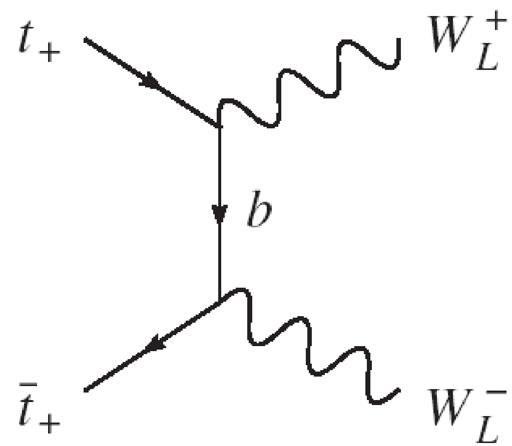


Helicities and colors are summed over for a stronger bound:

$$|\psi\rangle = \frac{1}{\sqrt{6}}\Big(|\bar{t}_{1+}t_{1+}\rangle + |\bar{t}_{2+}t_{2+}\rangle + |\bar{t}_{3+}t_{3+}\rangle - |\bar{t}_{1-}t_{1-}\rangle - |\bar{t}_{2-}t_{2-}\rangle - |\bar{t}_{3-}t_{3-}\rangle \Big)$$

AC Bound 2





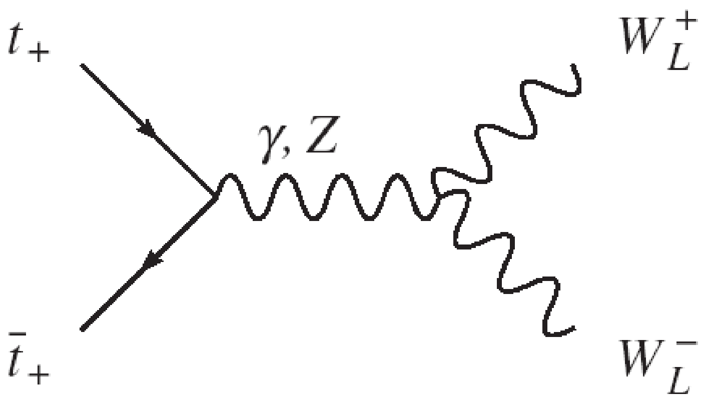
Leading order expressions in $M_W^2, m_t^2/s$ are used:

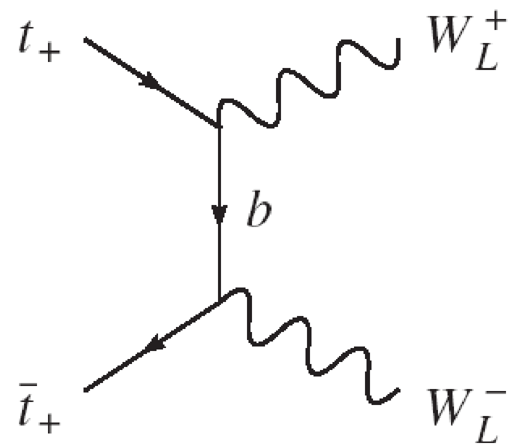
$$\epsilon^{\mu}_{W_L} \simeq \frac{k^{\mu}_{W_L}}{M_W}$$

$$\bar{v}_+ \left(\not{k}_1 - \not{k}_2\right) \left(g_L P_L + g_R P_R\right) u_+ \simeq m_t \sqrt{s}\ \cos\left(g_L + g_R\right)$$

$$\bar{v}_+ \not{k}_2 \left(\not{p}_1 - \not{k}_1\right) \not{k}_1 g_L P_L u_+ \simeq \frac{m_t\ t\sqrt{s}}{2} \left(1 + \cos\theta\right) g_L$$

AC Bound 3



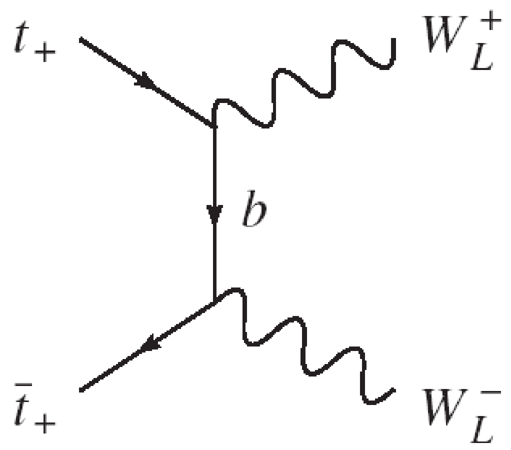


$$\mathcal{M} \simeq \frac{\sqrt{6s}\; m_t \cos\theta}{2M_W^2}\left(2g_{tt\gamma}g_{\gamma WW} + g_{LttZ}g_{ZWW} + g_{RttZ}g_{ZWW} - g_{LtbW}^2\right)$$
$$+\frac{\sqrt{6s}\; m_t}{2M_W^2}g_{LtbW}^2$$

The contribution from the gauge bosons cancels with part of the contribution from the b quark:

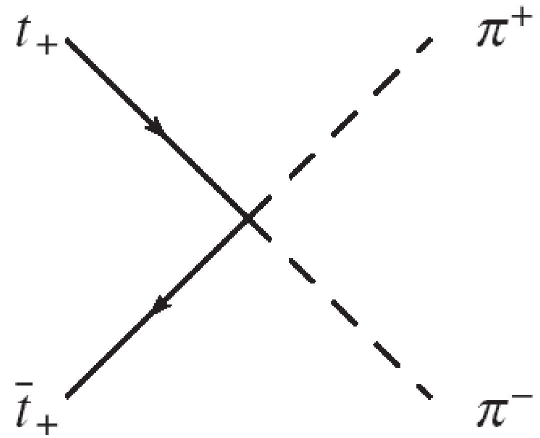
$$2g_{tt\gamma}g_{\gamma WW} + g_{LttZ}g_{ZWW} + g_{RttZ}g_{ZWW} - g_{LtbW}^2 = 0$$

AC Bound 4



$$\mathcal{M} \simeq \frac{\sqrt{6s}\; m_t}{2M_W^2} g_{LtbW}^2 = \frac{\sqrt{6s}\; m_t}{v^2}$$

AC Bound 5



Only the 4 point vertex contributes at order $\sqrt{s}$.

$$\mathcal{M} \simeq \sqrt{6s}\ g_{tt\pi\pi} = \frac{\sqrt{6s}\ m_t}{v^2}$$

AC Bound 6

$$a_0 = \frac{1}{32\pi}\int_{-1}^{1} d\cos\theta \; \mathcal{M} < \frac{1}{2}$$

$$a_0 \sim \frac{m_t\sqrt{6s}}{16\pi v^2}$$

$$\sqrt{s} \lesssim \frac{8\pi v^2}{m_t\sqrt{6}} \sim 3.5\mathrm{TeV}$$

- The J=0 partial wave amplitude is calculated.
- The real part must be less than ½ for unitarity.
- This gives the Appelquist-Chanowitz bound.

AC Bound: Golden

M. Golden: PLB **338**, 295 (1994)

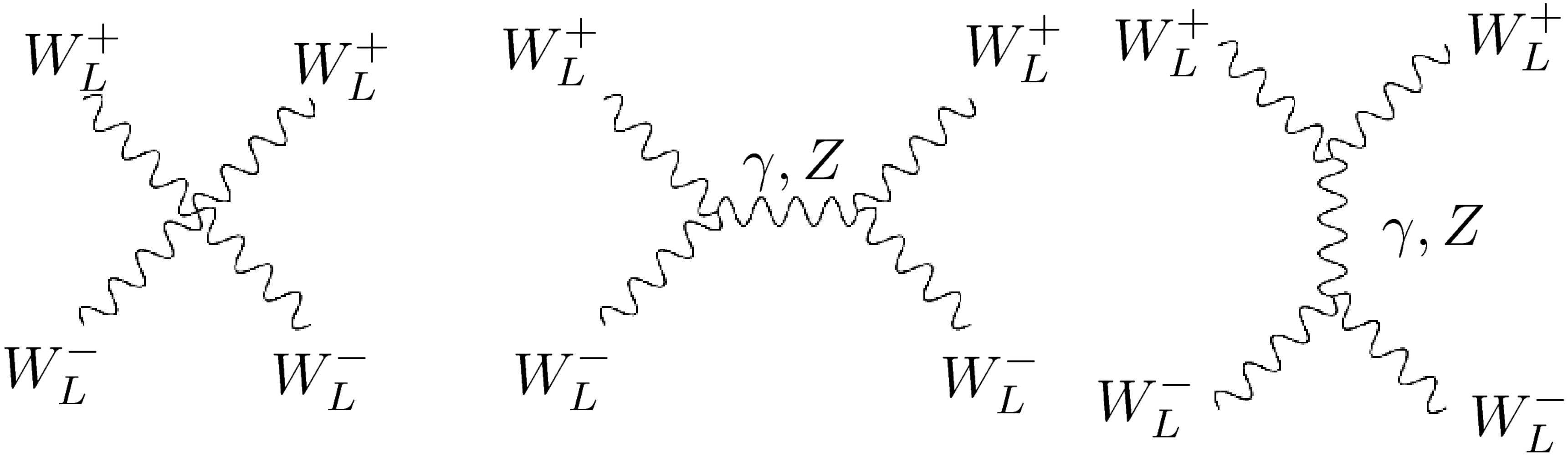
- We know that unitarity breaks down in the channel

  $$W_L^+ W_L^- \to W_L^+ W_L^-$$

- Some new physics has to appear before ~1TeV to unitarize WW scattering.
- Won't the fields that unitarize $W_L^+ W_L^- \to W_L^+ W_L^-$ also unitarize $t_+ \bar{t}_+ \to W_L^+ W_L^-$?
- Consider the Higgs:

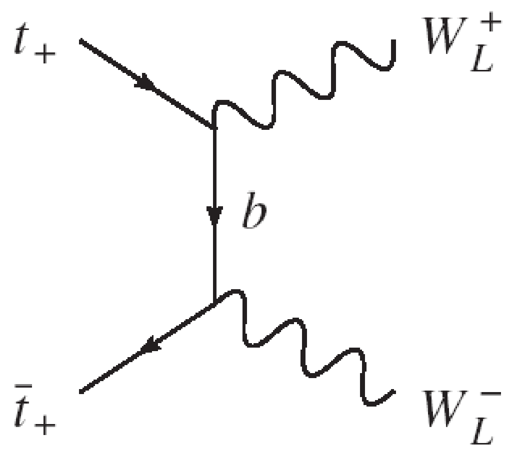
  It unitarizes both processes.

AC Bound: WW scattering



- This process becomes nonunitary at $\sqrt{s} \sim 1 TeV$.
- New scalar fields could help unitarize this process.
- New vector fields could help unitarize this process.
- New fermions could ***not*** help unitarize this process.

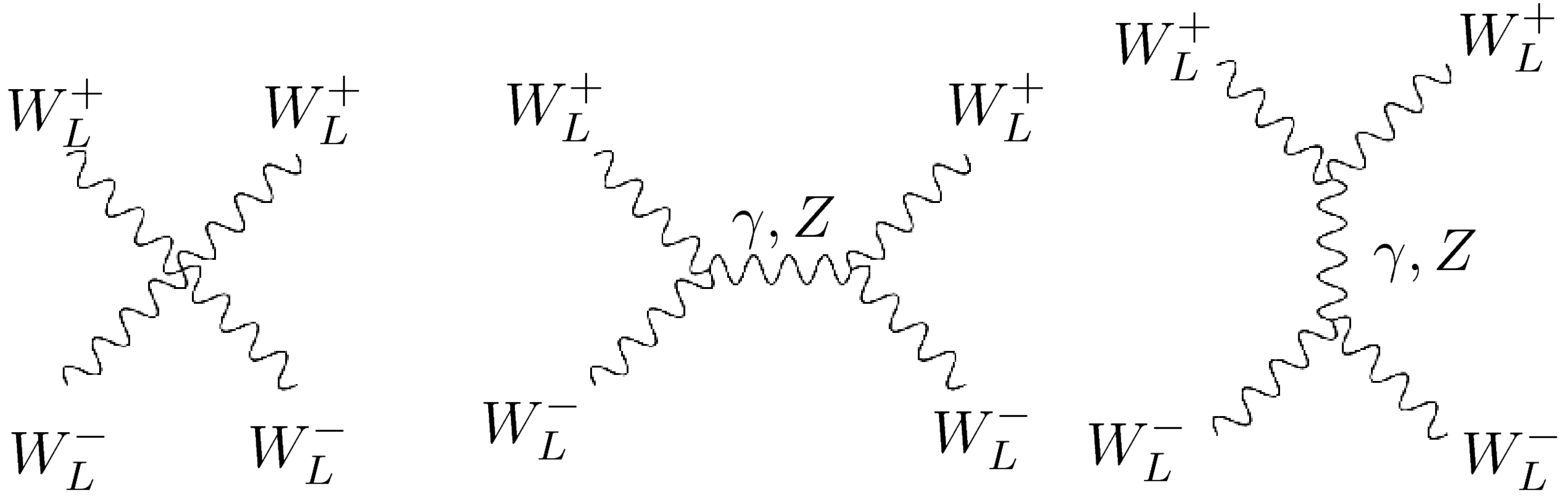
AC Bound: J=0 $t_+\bar{t}_+ \rightarrow W_L^+ W_L^-$



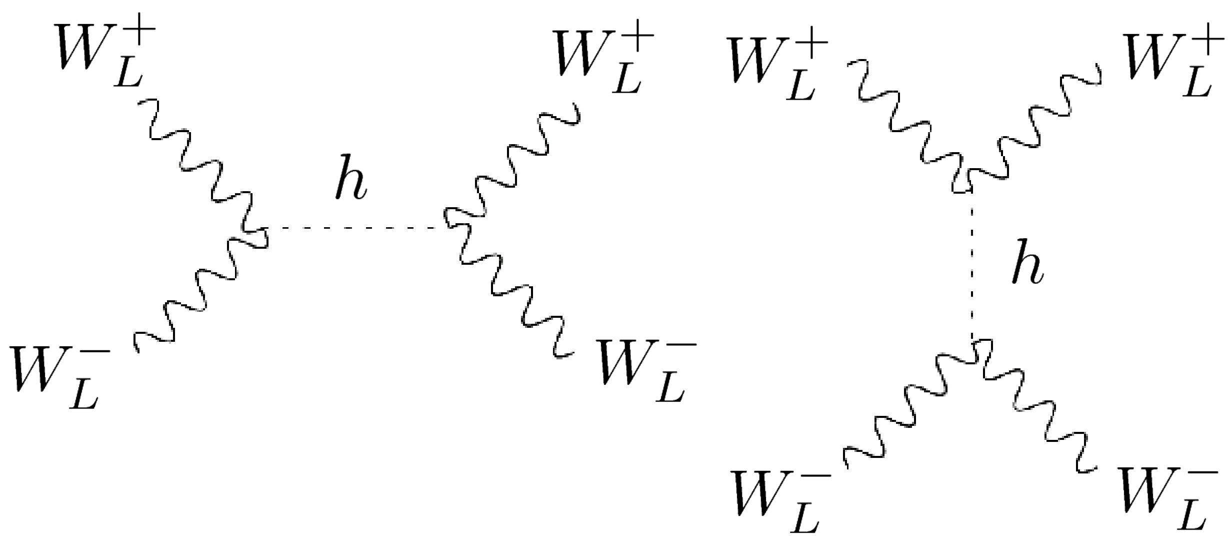
- This process becomes nonunitary at $\sqrt{s} \sim 3.5 TeV$.
- New scalar fields could help unitarize this process.
- New fermions could help unitarize this process.
- New vector fields could ***not*** help unitarize this process.

  (Vector fields in the S channel do not contribute to J=0.)

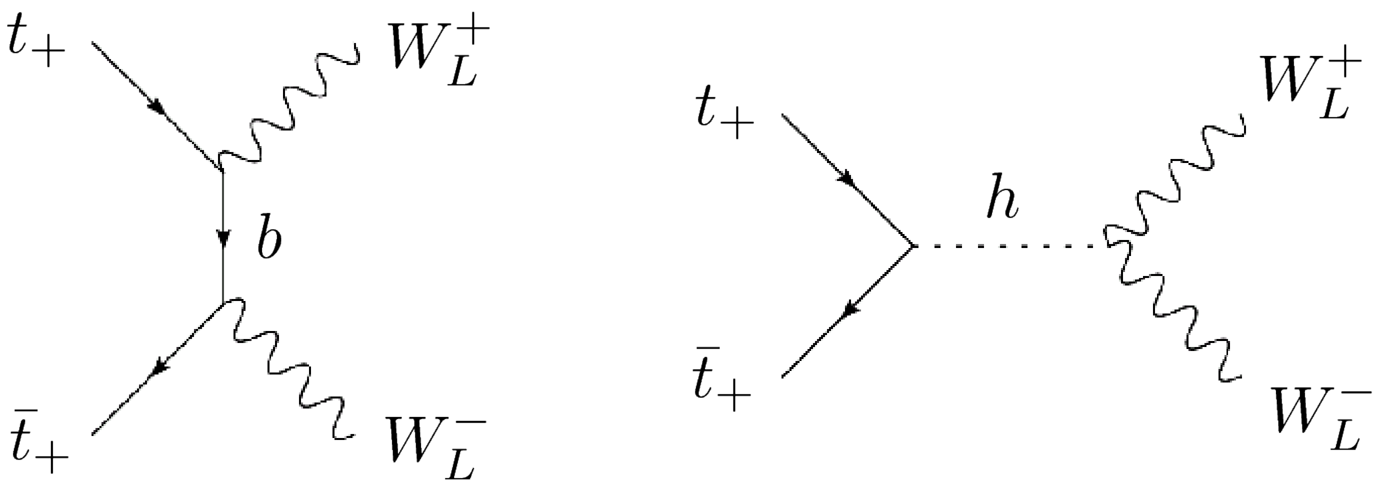
AC Bound: The Higgs



•A scalar field has the potential of unitarizing both WW scattering and ...



AC Bound: The Higgs

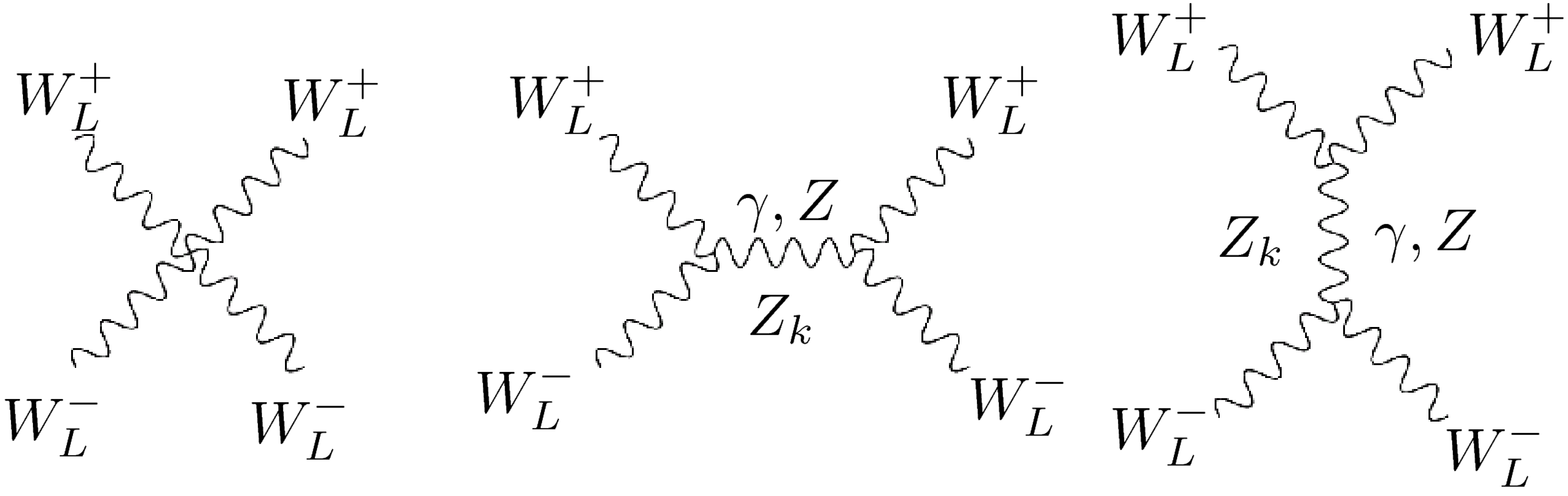


- But, in a Higgsless model, there are no scalars.
- A viable Higgsless model must:

  Unitarize $W_L^+ W_L^- \to W_L^+ W_L^-$ with gauge bosons.

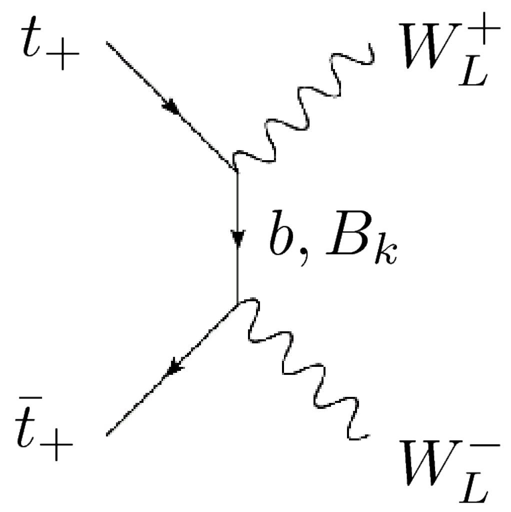
  Unitarize $t_+ \bar{t}_+ \to W_L^+ W_L^-$ with fermions.

Higgsless Unitarity of WW Scattering



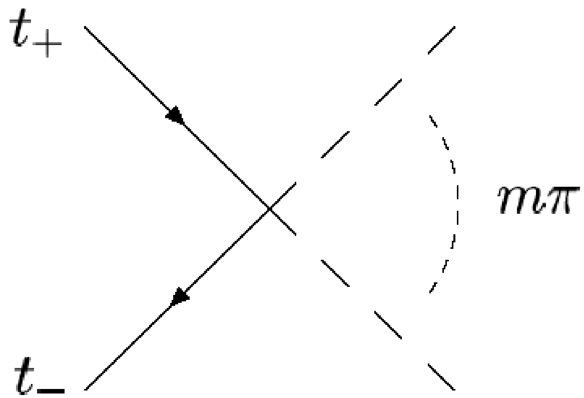
- WW scattering is unitarized by exchange of an infinite tower of Kaluza-Klein modes of the Z boson.
- PLB **525**, 175 (2002), PLB **532**, 121 (2002), PLB **562**, 109 (2003), IJMPA **20**, 3362 (2005)

Higgsless Unitarity of $t_+\bar{t}_+ \to W_L^+ W_L^-$



- $t_+\bar{t}_+ \to W_L^+ W_L^-$ in the J=0 channel, is unitarized by the exchange of an infinite tower of Kaluza-Klein modes of the bottom quark.
- Phys. Rev. D **75**, 073018 (2007)

$2 \to m$: MNW

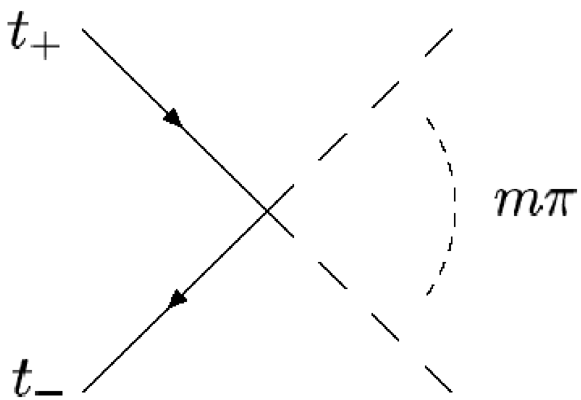


$$\mathcal{L} = -m_t \left(\bar{t}_L \ , \ 0 \right) e^{i \frac{2\pi}{v}} \begin{pmatrix} t_R \\ 0 \end{pmatrix}$$

$$g_{tt\pi^m} \sim \frac{m_t}{v^m}$$

- PRD **65**, 033004 (2002): Maltoni, Niczyporuk, and Willenbrock
- They noticed that $2 \to m$ may give a stronger bound than $2 \to 2$.
- They estimated $g_{tt\pi^m}$.

$2 \to m:$ MNW

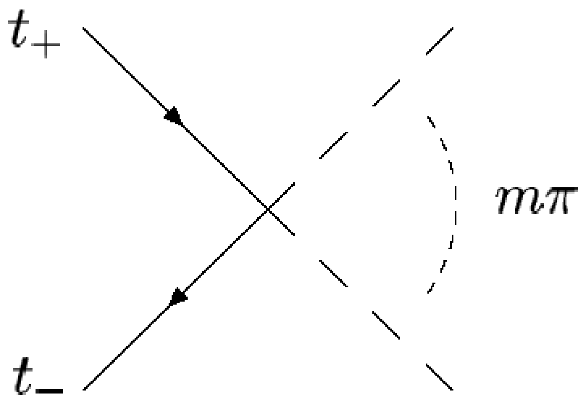


$$\mathcal{I}_m = \int \frac{d^3k_1 \cdots d^3k_m}{2E_1 \cdots 2E_m} \delta^{(4)}(P - k_1 - \cdots - k_m)$$

$$\sim s^{m-2}$$

- They estimated the phase space.

$2 \to m$: MNW

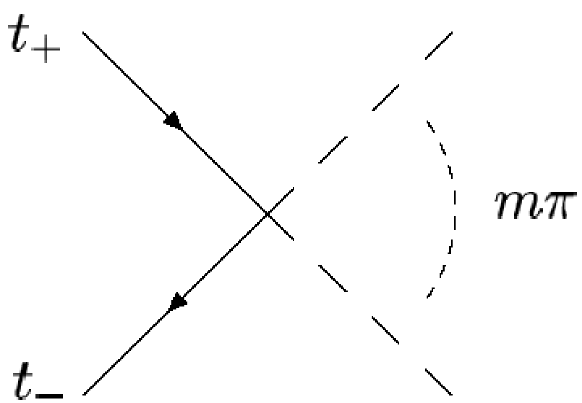


$$\sigma \sim \left(\frac{m_t}{v^m}\right)^2 s^{m-2} \lesssim \frac{4\pi}{s}$$

$$\sqrt{s} \lesssim \left(\frac{v^m}{m_t}\right)^{\frac{1}{m-1}} \underset{m\to\infty}{\longrightarrow} v$$

•Putting these together, they found that unitarity was bounded by a scale that approached v as the number of final states approached ∞.

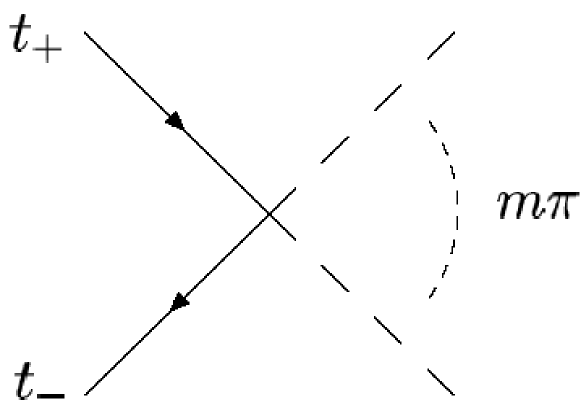
$2 \to m:$ DH



$$\sqrt{s} >\sim mM_W$$

- PRD **71**, 093009 (2005): Dicus and He
- Shouldn't there be at least enough energy to produce the final state particles?

$2 \to m$: DH

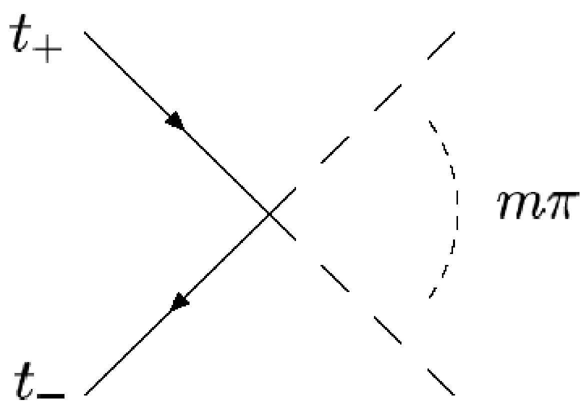


$$\mathcal{I}_m = \int \frac{d^3k_1 \cdots d^3k_m}{2E_1 \cdots 2E_m} \delta^{(4)}(P - k_1 - \cdots - k_m)$$

$$= \left(\frac{\pi}{2}\right)^{m-1} \frac{s^{m-2}}{(m-1)!(m-2)!}$$

•They carefully calculated the phase space and found the important factors (m-1)!(m-2)! in the denominator.

$2 \to m$: DH



$$\sigma \sim \left(\frac{m_t}{v^m}\right)^2 \frac{s^{m-2}}{(m-1)!(m-2)!} \lesssim \frac{4\pi}{s}$$

$$\sqrt{s} \lesssim \left(\frac{v^m}{m_t}\right)^{\frac{1}{m-1}} \Big((m-1)!(m-2)!\Big)^{\frac{1}{2(m-1)}}$$

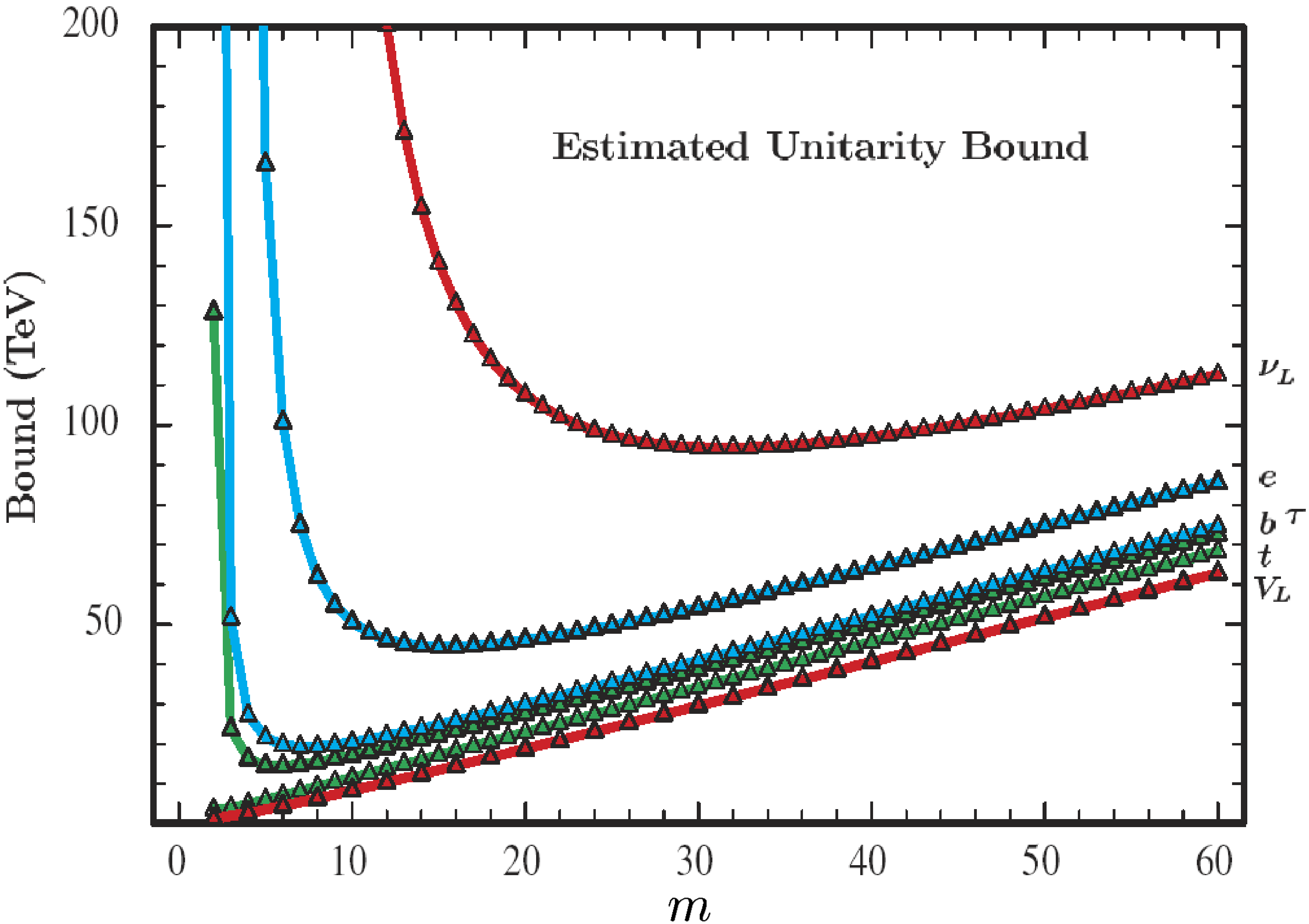
$$\overrightarrow{m\to\infty} \; \sqrt{s} \lesssim \frac{m}{3} v$$

•They found that the unitarity bound actually approaches mM_W as the number of final states approaches ∞ .

•For some particles, the bound does become stronger with increased number of final states.

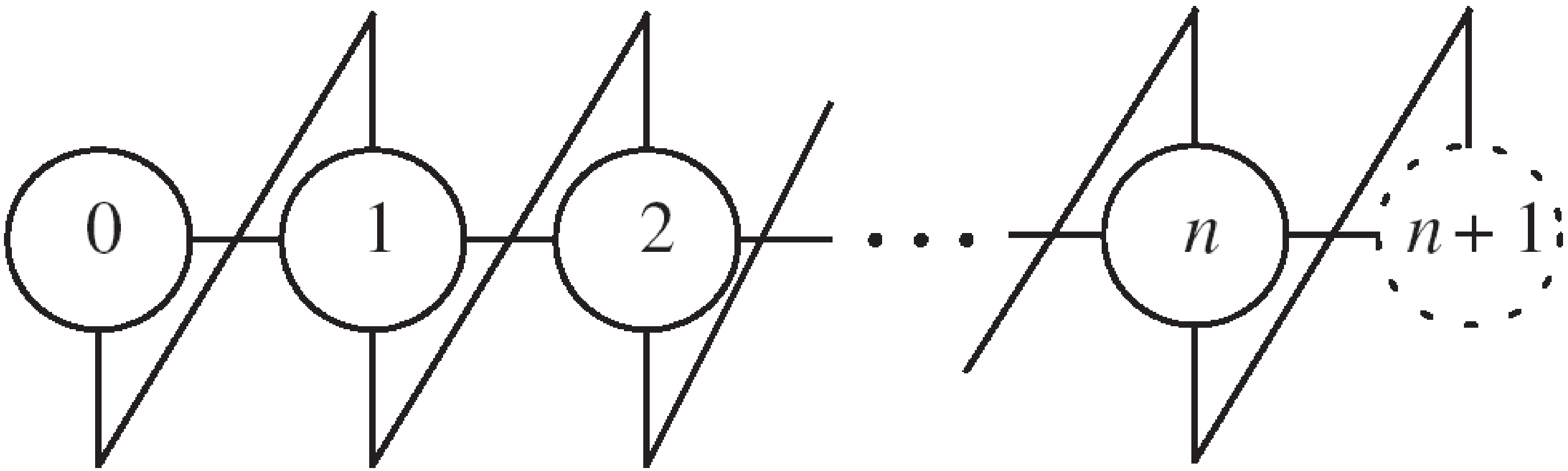
•However, they found that for the top quark, the $2 \to 2$ process still gives the strongest bound.

$2 \to m$: DH

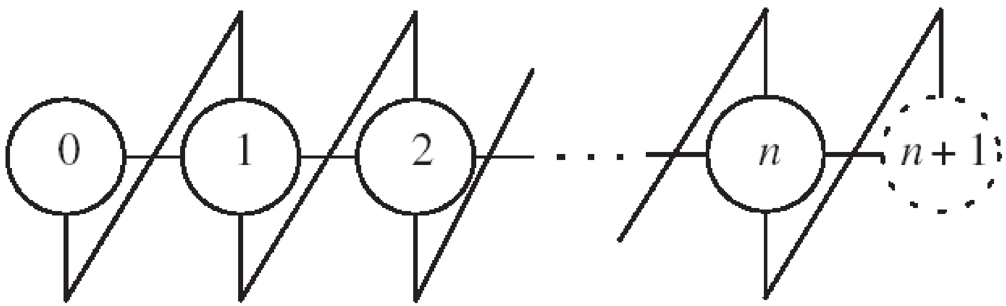


Estimated Unitarity Bound
Bound (TeV)
200
150
100
50
0
10
20
30
40
50
60
m
ν_L
e
b τ
t
V_L

n(+2) Site Model: Introduction



n(+2) Site Model: Gauge



$$\mathcal{L}_{F^2} = -\frac{1}{2}\mathrm{Tr}\left[\Sigma_j F_j^2\right]$$

where

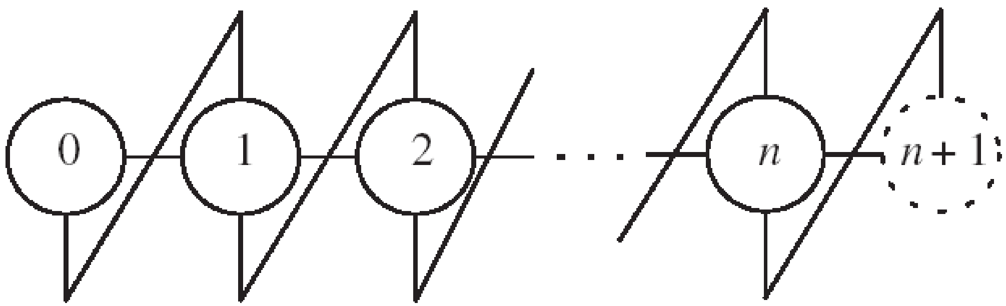
$$F_j^{\mu\nu} = \partial^\mu W_j^\nu - \partial^\nu W_j^\mu + ig\left[W_j^\mu, W_j^\nu\right]$$

$$F_{n+1}^{\mu\nu} = \partial^\mu W_{n+1}^\nu - \partial^\nu W_{n+1}^\mu$$

$$W_j = \begin{pmatrix} \frac{1}{2}W_j^0 & \frac{1}{\sqrt{2}}W_j^+ \\ \frac{1}{\sqrt{2}}W_j^- & -\frac{1}{2}W_j^0 \end{pmatrix}$$

$$W_{n+1} = \begin{pmatrix} \frac{1}{2}W_{n+1}^0 & 0 \\ 0 & -\frac{1}{2}W_{n+1}^0 \end{pmatrix}$$

n(+2) Site Model: Gauge-Goldstone



$$\mathcal{L}_{D\Sigma} = \frac{f^2}{2} \mathrm{Tr}\left[(D_\mu \Sigma_j)^\dagger D^\mu \Sigma_j \right]$$

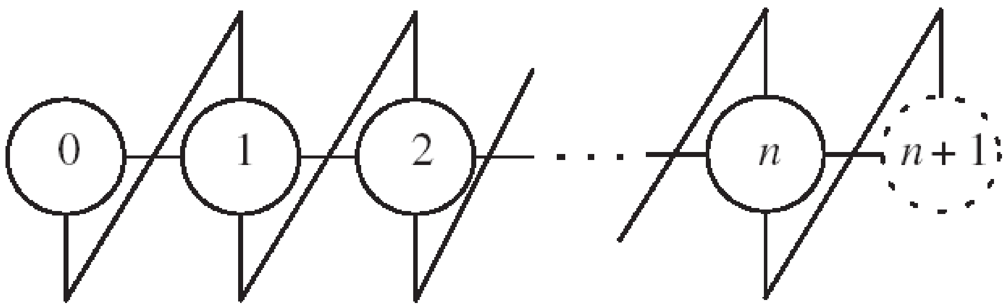
where

$$\Sigma_j = e^{i \frac{2\pi_j}{f}}$$

$$D_\mu \Sigma_j = \partial_\mu \Sigma_j + i g_j W_j \Sigma_j - i g_{j+1} \Sigma_j W_{j+1}$$

$$\pi_j = \begin{pmatrix} \frac{1}{2}\pi_j^0 & \frac{1}{\sqrt{2}}\pi_j^+ \\ \frac{1}{\sqrt{2}}\pi_j^- & -\frac{1}{2}\pi_j^0 \end{pmatrix}$$

n(+2) Site Model: Gauge Bosons



$$\mathcal{L}_{WW} = \frac{f^2}{2}\mathrm{Tr}\Big[\,(D_\mu I)^\dagger\, D^\mu I\Big]$$

$$M_n^2 = \frac{\tilde{g}^2 f^2}{4}\begin{pmatrix} x^2 & -x & 0 & 0 & \cdot & 0 & 0 \\ -x & 2 & -1 & 0 & \cdot & 0 & 0 \\ 0 & -1 & 2 & -1 & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & -1 & 0 \\ 0 & 0 & 0 & \cdot & -1 & 2 & -xt \\ 0 & 0 & 0 & \cdot & 0 & -xt & x^2t^2 \end{pmatrix}$$

$$M_{Z0} = \frac{gf}{2c\sqrt{n+1}}$$

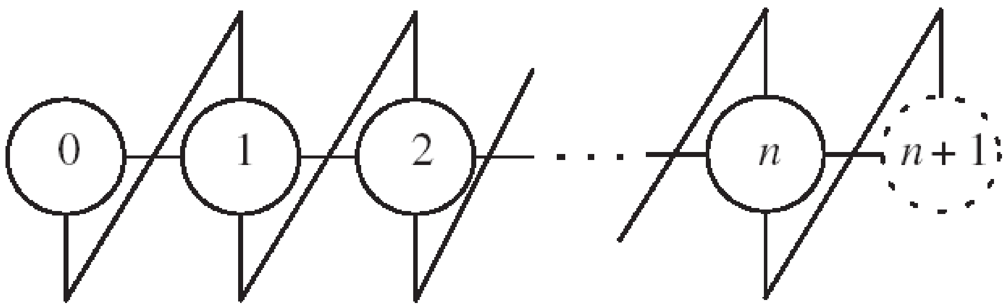
$$v_{Z0}^0 = c$$

$$v_{Z0}^j = \frac{c(n+1) - j/c}{n+1}x$$

$$v_{Z0}^{n+1} = -s$$

$$M_{W0} = \frac{gf}{2\sqrt{n+1}} \qquad v_{W0}^0 = 1 \qquad v_{W0}^j = \frac{n-j+1}{n+1}x$$

n(+2) Site Model: Fermion-Gauge



$$\mathcal{L}_{D\psi} = \Sigma_j \bar{\psi}_j \not{D} \psi_j$$

where

$$D_\mu \psi_j = \partial_\mu \psi_j + i g_j W_j \psi_j + i g' Y_{jf} W_1 \psi_j$$

$$D_\mu \psi_{R,n+1} = \partial_\mu \psi_{R,n+1} + i g' Y_{n+1,f} W_1 \psi_{R,n+1}$$

$Y_{jQ} = 1/3$

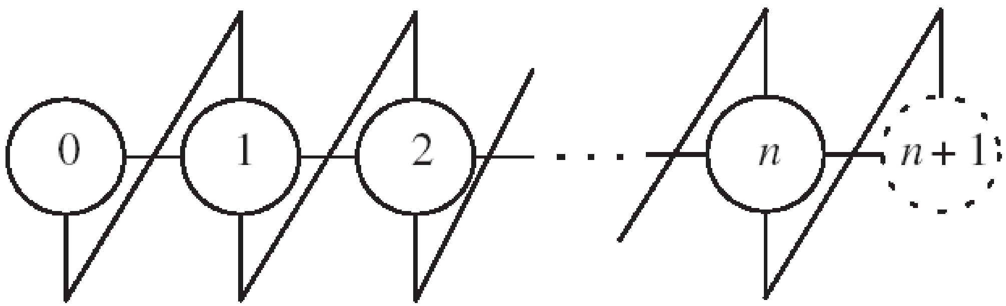
$Y_{n+1,u} = 4/3$

$Y_{n+1,d} = -2/3$

$Y_{jL} = -1$

$Y_{n+1,e} = -2$

n(+2) Site Model: Fermion-Goldstone



$$\mathcal{L}_{\psi\Sigma} = -M_F\Big[\epsilon_L \bar{\psi}_{L0}\Sigma_0\psi_{R1} - \sum_j \bar{\psi}_{Lj}\psi_{Rj} + \sum_j \bar{\psi}_{Lj}\Sigma_j\psi_{R,j+1} + \bar{\psi}_{Ln}\epsilon_R\Sigma_n\psi_{R,n+1} + \text{H.c.}\Big]$$

$$M_{F_0} = M_F\epsilon_L\epsilon_{R_f}$$

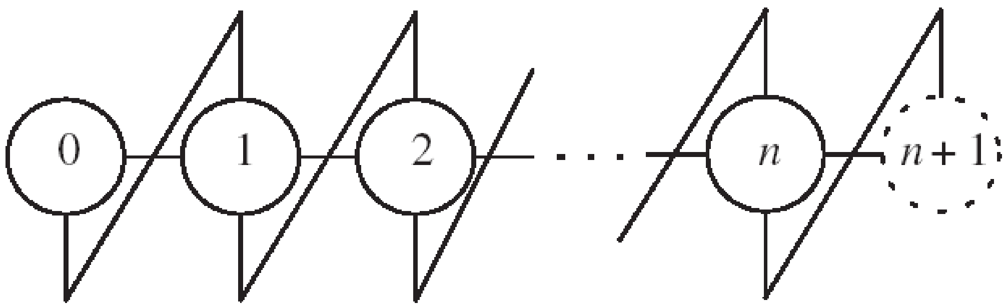
$$v^0_{LF_0} = 1$$

$$v^j_{LF_0} = \epsilon_L$$

$$v^j_{RF_0} = \epsilon_{R_f}$$

$$v^{n+1}_{RF_0} = 1$$

n(+2) Site Model: S



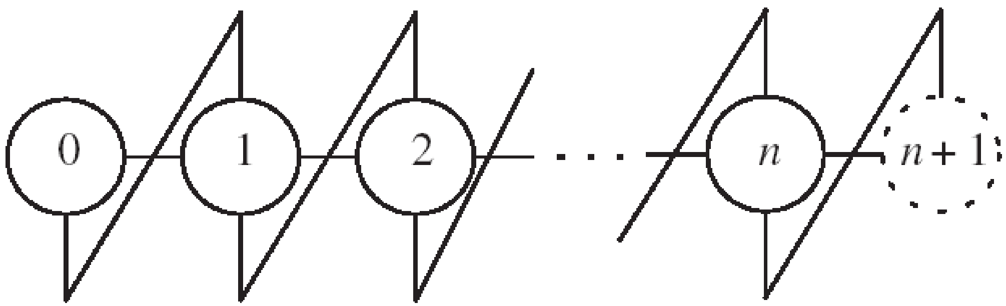
$$g_{We\nu} = \frac{e}{s_M}\left(1 + \frac{\alpha}{4s_M^2} S\right)$$

- Alterations of $g_{We\nu}$ can be parametrized by S.
- S can be calculated in the n(+2) site model and set to zero.

$$g_{We\nu} = \frac{e}{s_M}\left(1 + \frac{n(n+2)}{6(n+1)}x^2 - \frac{n}{2}\epsilon_L^2\right)$$

$$S = 0 \implies \epsilon_L^2 = \frac{n+2}{3(n+1)}x^2$$

n(+2) Site Model: Goldstone Bosons



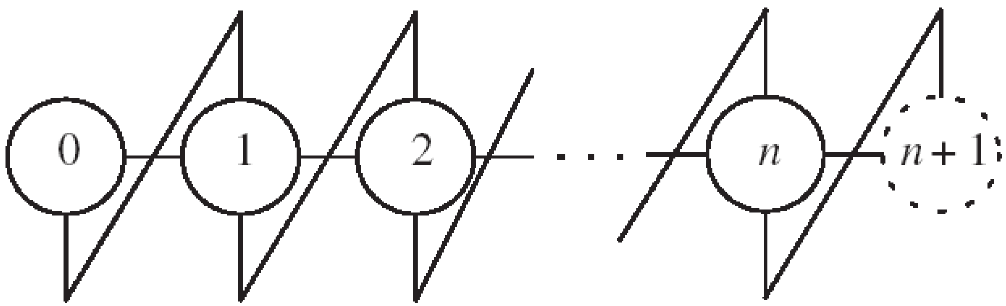
$$\mathcal{L}_{D\Sigma} = \frac{f^2}{4}\,\mathrm{Tr}\left[\sum_j (D_\mu \Sigma_j)^\dagger D^\mu \Sigma_j\right]$$

- The Goldstone bosons are determined by their mixing with the gauge bosons that eat them.
- The Goldstone bosons eaten by the W and Z are particularly simple.

$$\mathcal{L}_{\pi W} = -i\frac{\tilde{g}f}{2}\Bigg[\{\partial_\mu \pi_0, xW_0^\mu - W_1^\mu\} + \sum_{j=1}^{n-1}\{\partial_\mu \pi_j, W_j^\mu - W_{j+1}^\mu\} + \{\partial_\mu \pi_n, W_n^\mu - xtW_{n+1}^\mu\}\Bigg]$$

$$v_{\pi_0^\pm}^{[l]} = \frac{1}{\sqrt{n+1}} = v_{\pi_0^0}^{[l]}$$

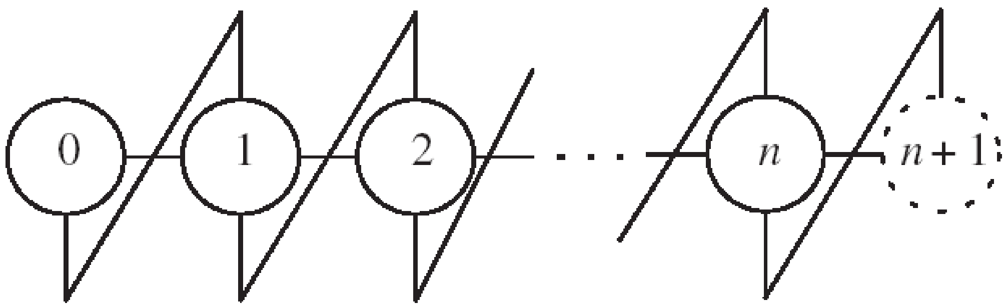
n(+2) Site Model: Couplings



$$\mathcal{L}_{\psi\Sigma} = -M_F\Big[\epsilon_L \bar{\psi}_{L0}\Sigma_0\psi_{R1} - \sum_j \bar{\psi}_{Lj}\psi_{Rj} + \sum_j \bar{\psi}_{Lj}\Sigma_j\psi_{R,j+1} + \bar{\psi}_{Ln}\epsilon_R\Sigma_n\psi_{R,n+1} + \text{H.c.}\Big]$$

$$g_{RtF_k\pi} = -i\frac{\sqrt{2}M_F}{f}\Big[\epsilon_L v^0_{LF_k} v^1_{Rt} v^{[0]}_\pi + \sum_i v^i_{LF_k} v^{i+1}_{Rt} v^{[i]}_\pi + \epsilon_{Rt} v^n_{LF_k} v^{n+1}_{Rt} v^{[n]}_\pi\Big] = \frac{i\sqrt{2}M_F\epsilon_R}{\sqrt{2n+1}(n+1)v}\tan\Big[\frac{(n-k+1)\pi}{2n+1}\Big]$$

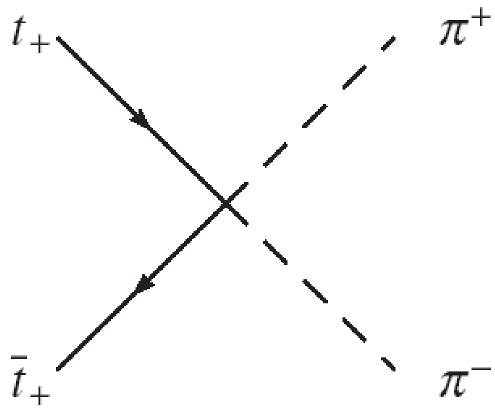
n(+2) Site Model: Couplings

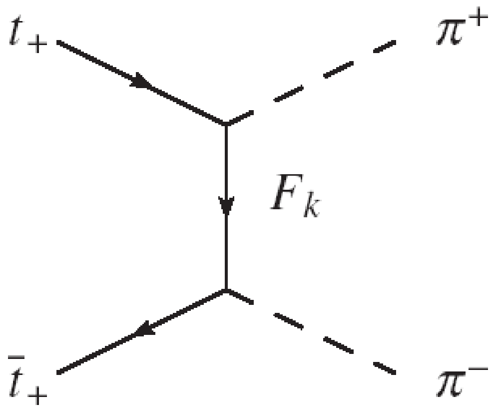


$$\mathcal{L}_{\psi\Sigma} = -M_F\Big[\epsilon_L \bar{\psi}_{L0}\Sigma_0\psi_{R1} - \sum_j \bar{\psi}_{Lj}\psi_{Rj} + \sum_j \bar{\psi}_{Lj}\Sigma_j\psi_{R,j+1} + \bar{\psi}_{Ln}\epsilon_R\Sigma_n\psi_{R,n+1} + \text{H.c.}\Big]$$

$$g_{tt\pi^+\pi^-} = \frac{M_F}{f^2}\Big[\epsilon_L v^0_{Lt} v^1_{Rt}(v^{[0]}_\pi)^2 + \sum_i v^i_{Lt} v^{i+1}_{Rt}(v^{[i]}_\pi)^2 + \epsilon_{Rt} v^n_{Lt} v^{n+1}_{Rt}(v^{[n]}_\pi)^2\Big] = \frac{m_t}{(n+1)v^2}.$$

n(+2) Site Model: Calculation





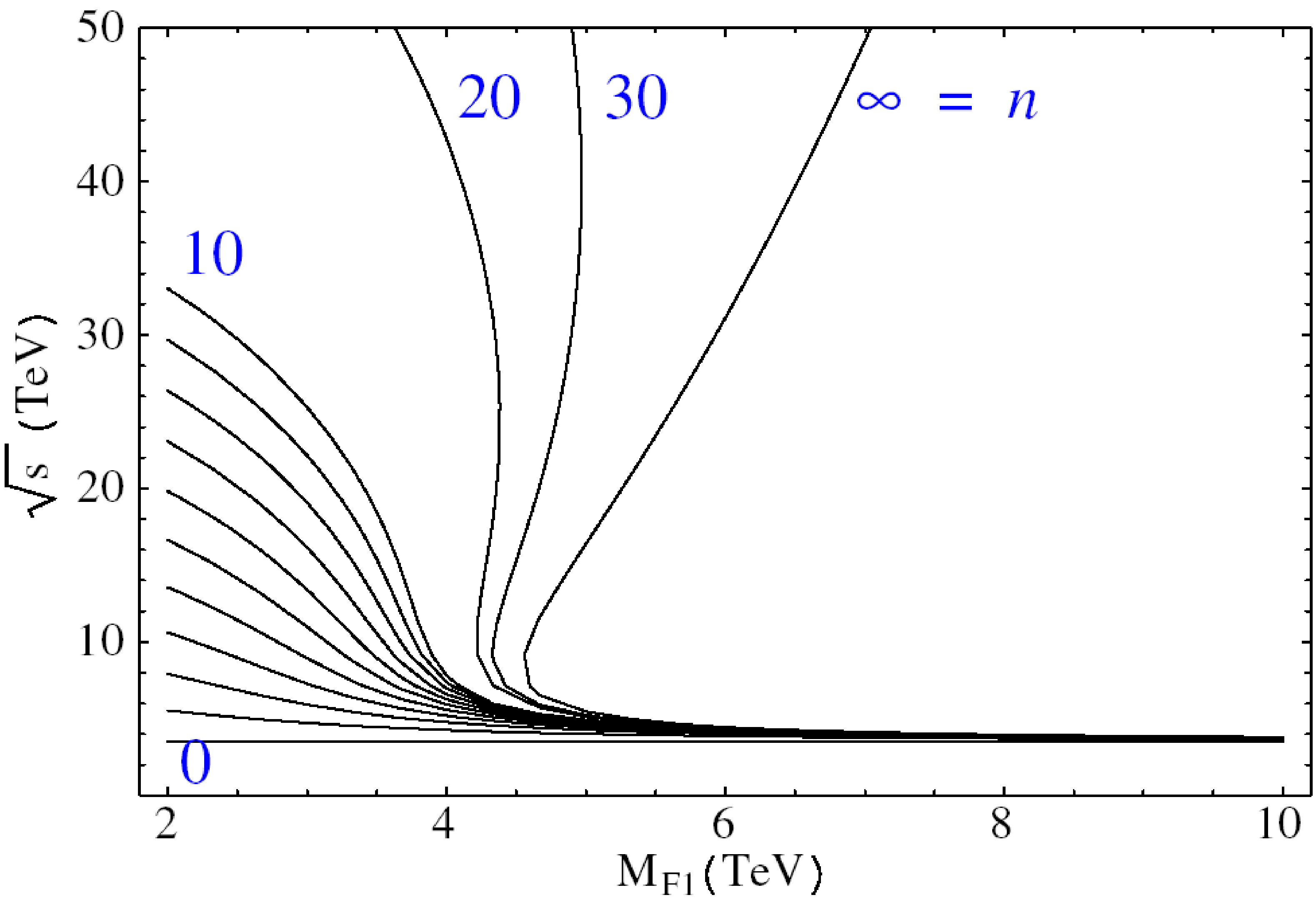
- The 4 point diagram grows like $\sqrt{s}$ for all energies.
- The T channel diagrams grow like $\sqrt{s}$ up to M_{F_k} .
- It is the F_k that unitarize this process and not the W_k !

$$\mathcal{M} = \sqrt{6s}\left(g_{tt\pi^+\pi^-} - \sum_k \frac{M_{F_k} g_{LtF_k\pi} g_{RtF_k\pi}}{t - M_{F_k}^2}\right)$$

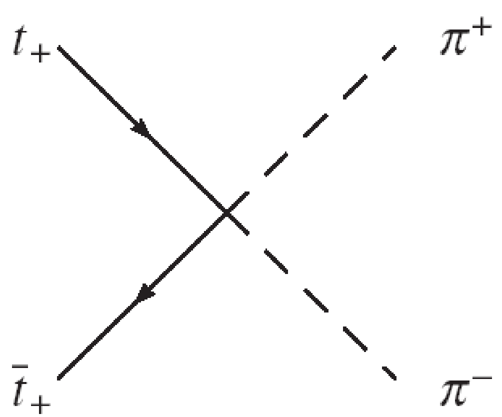
$$\begin{aligned} a_0 &= \frac{1}{32\pi}\int_{-1}^{1} d\cos\theta\, \mathcal{M} \\ &= \frac{\sqrt{6}}{16\pi}\left[g_{tt\pi^+\pi^-}\sqrt{s} + \sum_k g_{LtF_k\pi} g_{RtF_k\pi}\, g\left(\frac{\sqrt{s}}{M_{F_k}}\right)\right] \end{aligned}$$

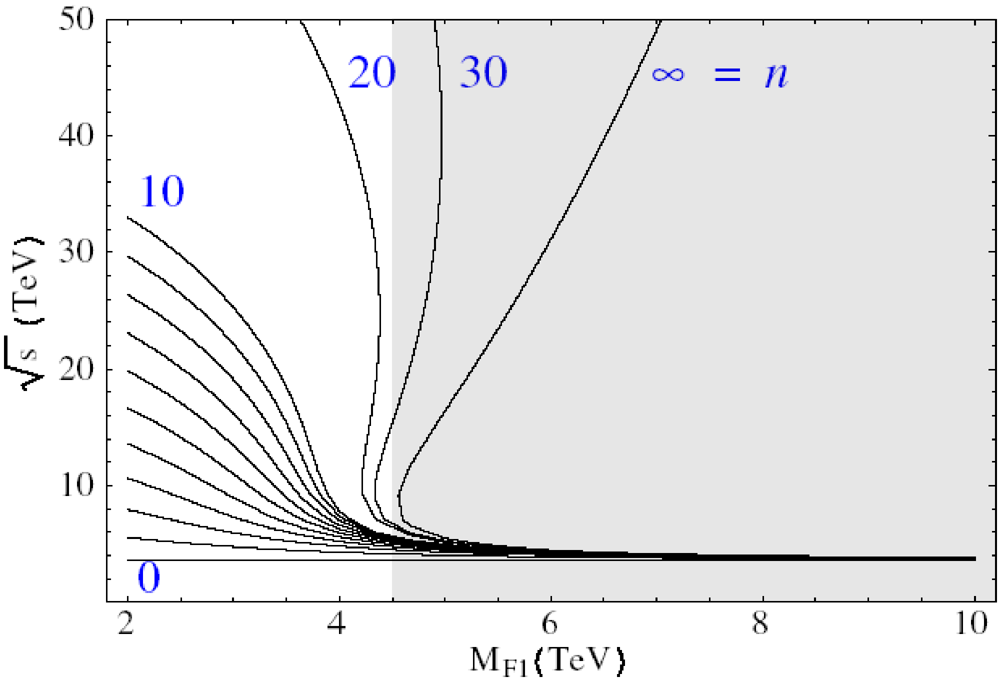
$$g(x) = \tfrac{1}{x}\ln(1 + x^2)$$

n(+2) Site Model: Unitarity Bound



n(+2) Site Model: $M_{F1} \ll 4.5\text{TeV}$





•For $M_{F1} \ll 4.5\text{TeV}$, the bound is determined by the 4 point vertex.

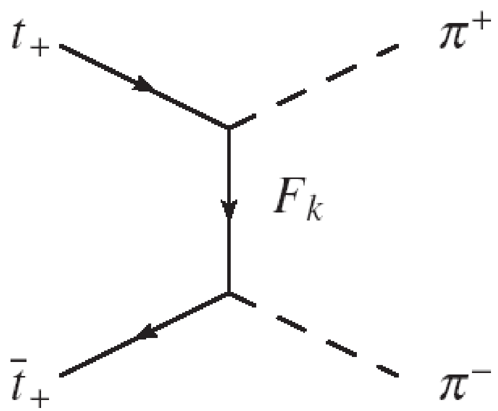
•In that limit, the bound is just a multiple of the AC bound.

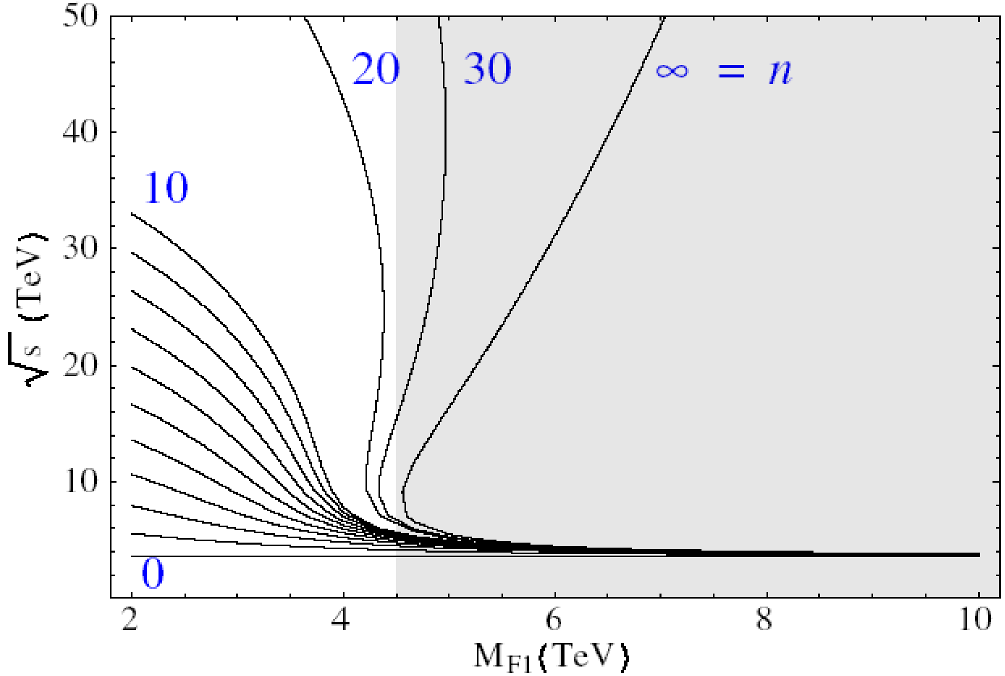
•The bound disappears in the continuum limit.

$$a_0 \simeq \frac{\sqrt{6s}m_t}{16\pi v^2(n+1)} \lesssim \frac{1}{2}$$

$$\sqrt{s} \lesssim (n+1)3.5 \text{ TeV}$$

n(+2) Site Model: $n \to \infty$





- The edge can be determined in the $n \to \infty$ limit where the 4 point vertex disappears
- The T channel is dominated by the first KK mode.

$$\lim_{n\to\infty} a_0 = \frac{2\sqrt{6}M_{F_1}m_t}{\pi^4 v^2}\sum_k \frac{(-1)^{k+1}}{(2k-1)^2}\, g\!\left(\frac{\sqrt{s}}{(2k-1)M_{F_1}}\right)$$

$$\lim_{n\to\infty} a_0(k=1) \approx \frac{2\sqrt{6}M_{F_1}m_t}{\pi^4 v^2}\, g\!\left(\frac{\sqrt{s}}{M_{F_1}}\right)$$

$$M_{F_1} \lesssim \frac{\pi^4 v^2}{2\sqrt{6}m_t \ln(5)} \sim 4.25\ \text{TeV}$$

Summary

In Higgsless models:

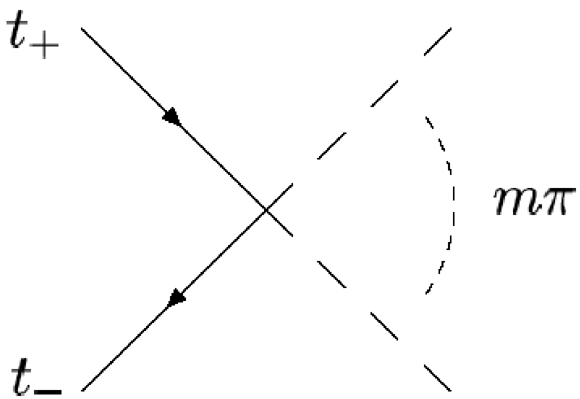
- The process $t_+\bar{t}_+ \to W_L^+ W_L^-$ is unitarized by B_k while $W_L^+ W_L^- \to W_L^+ W_L^-$ is unitarized by Z_k.

- The bound on the scale of fermion mass generation is independent of the scale of gauge boson mass generation.

- Even for a small number of new 'sites', the scale where new physics responsible for the mass generation of the fermions appears can be significantly altered and weakened by the presence of mixing between the fields of the different 'sites'.

Appendix

$2 \to m$: n(+2) site



$$\sim \frac{m_t}{(n+1)^{m-1} v^m} \xrightarrow[n\to\infty]{} 0$$

- These vertices are further suppressed by $1/n^m$.
- These vertices disappear as $n \to \infty$.

$$g_{tt\pi^m} = \frac{2^m M_F}{\left(\sqrt{2}\right)^m m! f^m} \left[\epsilon_L v_{Lt}^0 v_{Rt}^1 \left(v_\pi^0\right)^m + \sum_j v_{Lt}^j v_{Rt}^{j+1} \left(v_\pi^j\right)^m + \epsilon_{Rt} v_{Lt}^n v_{Rt}^{n+1} \left(v_\pi^n\right)^m \right]$$

$$g_{tt\pi^m} = \frac{\left(\sqrt{2}\right)^m m_t}{m!(n+1)^{m-1} v^m}$$